Statistics

Summer 2023

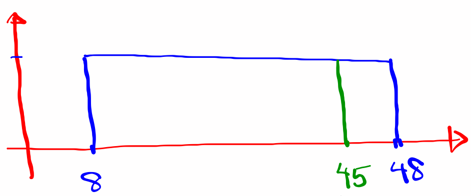
Lecture 12

Feb 19-8:47 AM

Consider a uniform Prob. dist from $\boxed{8}$ to $\boxed{48}$.

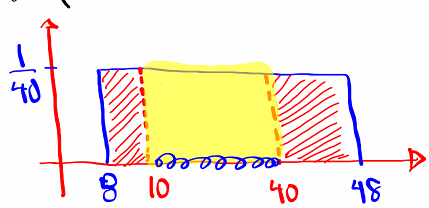
1) Draw & label.

$\frac{1}{48-8} = \frac{1}{40}$



2) $P(x=45) = 0$

3) $P(x<10 \text{ OR } x>40)$



$= 1 - P(10 < x < 40)$

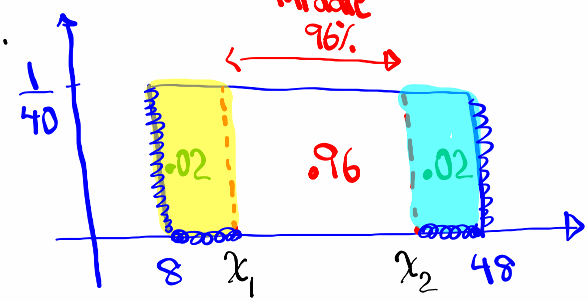
Total Area (Prob.)

$= 1 - (40-10) \cdot \frac{1}{40} = 1 - \frac{30}{40}$

$= \boxed{\frac{1}{4}}$

Jul 3-7:33 AM

4) Find two x-values that separate the middle 96% from the rest.



$1 - .96 = .04$

$.04 \div 2 = .02$

$(x_1 - 8) \cdot \frac{1}{40} = .02$

$x_1 - 8 = 40(.02)$

$x_1 - 8 = .8$

$\boxed{x_1 = 8.8}$

$(48 - x_2) \cdot \frac{1}{40} = .02$

$48 - x_2 = 40(.02)$

$48 - x_2 = .8$

$48 - .8 = x_2$

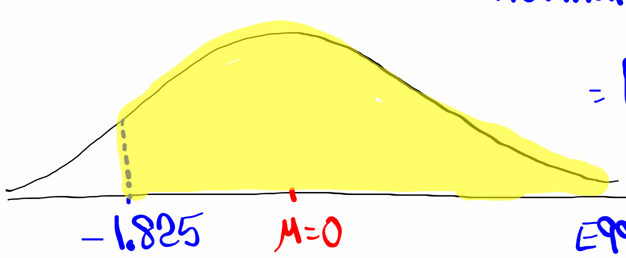
$\boxed{x_2 = 47.2}$

Jul 3-7:38 AM

Find $P(Z > -1.825)$

$= \text{normalcdf}(-1.825, E99, 0, 1)$

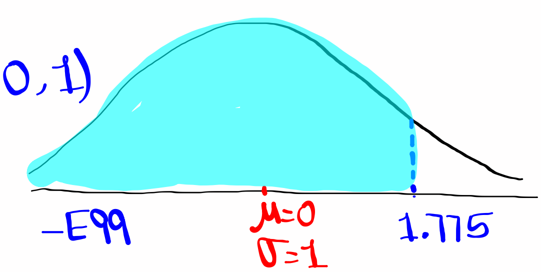
$= \boxed{.966}$



$P(Z < 1.775)$

$= \text{normalcdf}(-E99, 1.775, 0, 1)$

$= \boxed{.962}$



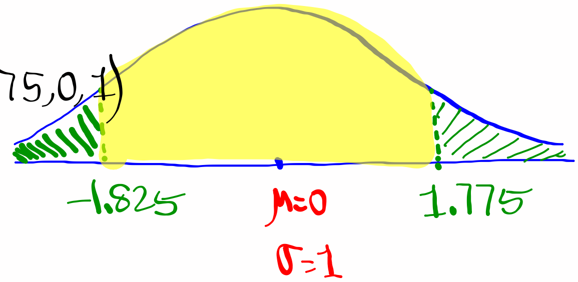
Jul 3-7:44 AM

Find $P(Z < -1.825 \text{ OR } Z > 1.775)$

$= 1 - P(-1.825 < Z < 1.775)$

$= 1 - \text{normalcdf}(-1.825, 1.775, 0, 1)$

$= \boxed{.072}$



If OR was And $\rightarrow P(Z < -1.825 \text{ AND } Z > 1.775)$

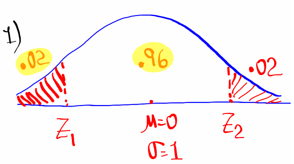
No overlapping (M.E.E.) $\rightarrow \boxed{0}$

Jul 3-7:50 AM

Find two Z-Values, round to 3-decimal places, that Separate the middle 96% from the rest.

$Z_1 = P_2 = \text{invNorm}(.02, 0, 1)$

$= \boxed{-2.054}$

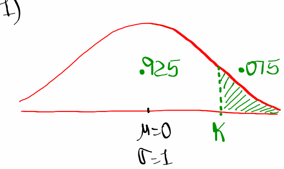


$Z_2 = P_{98} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$ ✓

Find K such that $P(Z > K) = .075$ Right Area

$K = \text{invNorm}(.925, 0, 1)$

$= \boxed{1.440}$



Find K such that

$P(Z < K) = .075$ Left Area

Ans. $\boxed{-1.440}$ by Symmetry.

Jul 3-7:54 AM

Find $Z = P_{95}$

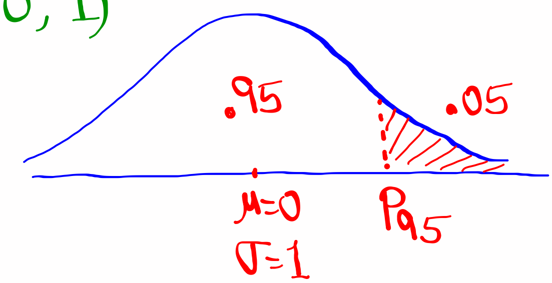
95% below & 5% above

Left Area & Right Area

.95 & .05

$Z = P_{95} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$



Jul 3-8:03 AM

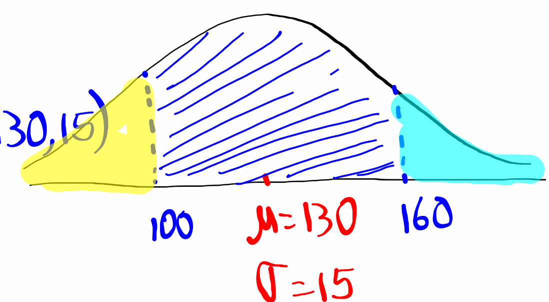
Consider a normal Prob. dist with $\mu = 130$ and $\sigma = 15$.

$N(130, 15)$

$P(100 < x < 160)$

$= \text{normalcdf}(100, 160, 130, 15)$

$= \boxed{.954}$ ✓



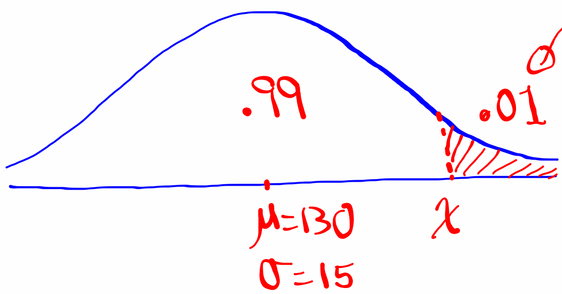
$P(x < 100 \text{ OR } x > 160) = 1 - P(100 < x < 160)$

$= 1 - .954 = \boxed{.046}$

IF it was AND → $\boxed{0}$

Jul 3-8:06 AM

Find x-value that separates the top 1% from the rest. Round to whole #.



$x = \text{invNorm}(.99, 130, 15)$

$= 164.895$

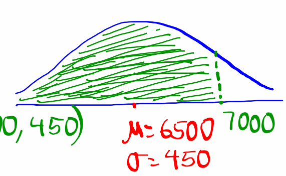
$= \boxed{165}$

Jul 3-8:11 AM

Salaries of nurses are normally dist. with mean of \$6500/mo. with standard deviation of \$450/mo. N(6500, 450)

If we randomly select 1 nurse, find the Prob. of he/she makes

a) below \$7000/mo.

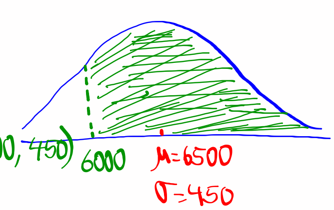


$P(x<7000)$

$= \text{normalcdf}(-E99, 7000, 6500, 450)$

$= \boxed{.867}$

b) above \$6000/mo.



$= \text{normalcdf}(6000, E99, 6500, 450)$

$= \boxed{.867}$

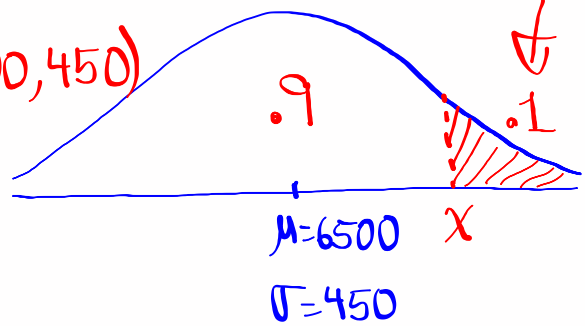
Jul 3-8:15 AM

Find a Salary, round to whole #, that Separates the $\boxed{\text{top } 10\%}$ from the rest.

$x = \text{invNorm}(.9, 6500, 450)$

$= 7076.698$

$\approx \$ \boxed{7077}$



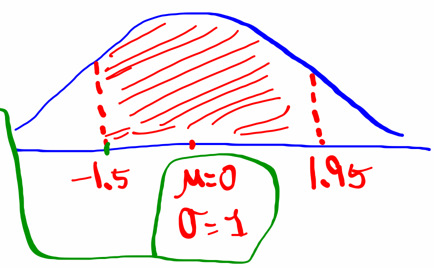
Jul 3-8:23 AM

Class QZ 14

1) Find $P(-1.5 < Z < 1.95)$

$= \text{normalcdf}(-1.5, 1.95, 0, 1)$

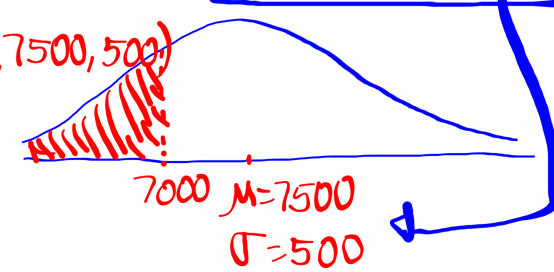
$= \boxed{.908}$ ✓



2) Find $P(x < 7000)$ given $\boxed{N(7500, 500)}$

$= \text{normalcdf}(-E99, 7000, 7500, 500)$

$= \boxed{.159}$ ✓



Jul 3-8:26 AM

Consider the population

2, 4, 6, 8

Store in L1, Use $\boxed{\text{1-VAR STATS}}$, find

$\mu = 5$ $\qquad \sigma = 2.236 \qquad \sigma^2 = 5$

take all Samples of size 2 with replacement.

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

Now let's find $\bar{x}$ of each Sample

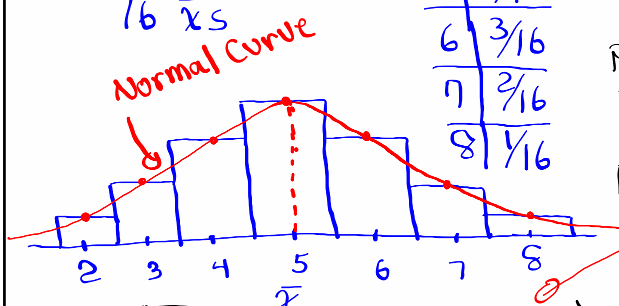
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

Jul 3-9:02 AM

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 $\bar{x}$s

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16



Place

$\bar{x} \to L2$,

$P(\bar{x}) \to L3$

list Freqlist

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3 to find

$\boxed{\mu_{\bar{x}} = 5}$ $\qquad \boxed{\sigma = 1.581} \qquad \boxed{\sigma^2_{\bar{x}} = 2.5 = \frac{5}{2}}$

Jul 3-9:10 AM

Consider a population below

2 4 6 8 10

Store in L1, Use $\boxed{\text{1-Var Stats}}$ with L1 to find

$\boxed{\mu = 6}$ $\boxed{\sigma = 2.828}$ $\boxed{\sigma^2 = 8}$

Now let's take all samples with size 2 with replacement from this population

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

Now find $\bar{x}$ of each sample.

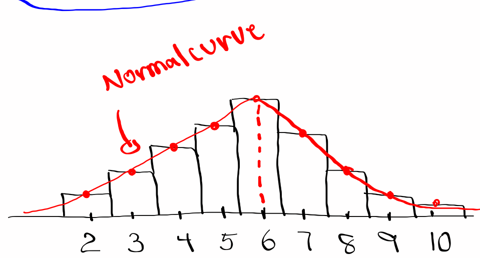
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

There are 25 $\bar{x}$s.

Jul 3-9:18 AM

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25



$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use $\boxed{\text{1-VAR Stats}}$ with L2 & L3, find

$\mu_{\bar{x}} = 6$ $\sigma = 2$ $\sigma_{\bar{x}}^2 = 4 = \frac{8}{2}$

Jul 3-9:26 AM

Repeat the entire last example for the population given below:

1 3 5 7 9

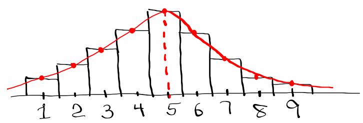
$\mu = 5$ $\sigma = 2.828$ $\sigma^2 = 8$

$\bar{x}$ of all Samples of Size 2 with replacement

1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

25 $\bar{x}$s

$\bar{x}$	$P(\bar{x})$
1	1/25
2	2/25
3	3/25
4	4/25
5	5/25
6	4/25
7	3/25
8	2/25
9	1/25



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3, find

$\mu_{\bar{x}} = 5$ $\sigma_{\bar{x}} = 2$ $\sigma^2_{\bar{x}} = 4 = \frac{8}{2}$

Jul 3-9:36 AM

Central Limit Theorem

(CLT)

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\boxed{\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Jul 3-10:04 AM

Suppose we have $N(120, 25)$

we take Samples of Size 4.

$\mu_{\bar{x}} = \mu = 120$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{25}{\sqrt{4}} = \dfrac{25}{2} = 12.5$

Consider all Samples of Size 16

From a $N(240, 20)$

$\mu_{\bar{x}} = \mu = \boxed{240}$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{20}{\sqrt{16}} = \dfrac{20}{4} = 5$

Jul 3-10:07 AM

Class QZ 15

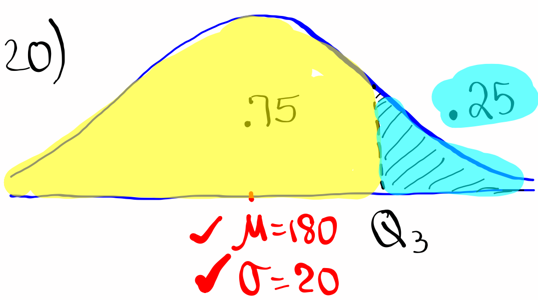
Given $\boxed{N(180, 20)}$ 75% below 25% above

Find $x = Q_3$, Round to a whole #.

$x = \boxed{\text{invNorm}}(.75, 180, 20)$

$= 193.490$

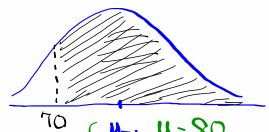
$\approx \boxed{193}$ ✓



Jul 3-10:10 AM

Exam Scores are normally dist. with
$\mu = 80$ and $\sigma = 10$. $N(80, 10)$

If we randomly select $\boxed{4 \text{ exams}}$ ($n=4$) find
the Prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) score is
above 70.



$P(\bar{x} > 70)$

$= \text{normalcdf}(70, E99, 80, 5)$

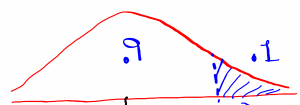
$= \boxed{.977}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 80 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{4}} = 5 \end{cases}$

For randomly select groups of 4, find $\bar{x} = P_{90}$
Round to whole #.

90% below,
10% above

$\bar{x} = \text{invNorm}(.9, 80, 5)$
$= 86.408$
$= \boxed{86}$



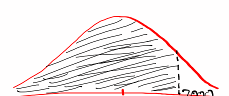
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 80 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{4}} = 5 \end{cases}$ $\bar{x} = P_{90}$

Jul 3-10:31 AM

Salaries of nurses are normally dist.
with $\mu = \$6500/\text{mo.}$ with $\sigma = \$400/\text{mo.}$
$N(6500, 400)$

If we randomly select $\boxed{5 \text{ nurses}}$ ($n=5$) find
the Prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) Salary is
below \$7000/mo.

$P(\bar{x} < 7000)$



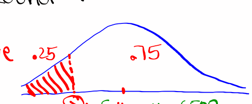
$= \text{normalcdf}(-E99, 7000, 6500, 400/\sqrt{5})$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6500 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{400}{\sqrt{5}} \end{cases}$

$= \boxed{.997}$

For randomly Selected $\boxed{\text{group of } 5}$ nurses,
find $\bar{x} = Q_1$, Round to whole #.

25% below
75% above



$\bar{x} = Q_1 = \text{invNorm}(.25, 6500, 400/\sqrt{5})$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6500 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{400}{\sqrt{5}} \end{cases}$

$= 6379.344$

$\approx \$\boxed{6379}$

Jul 3-10:39 AM

Credit Scores are normally dist with the mean of 700 and standard deviation of 65. $N(700, 65)$

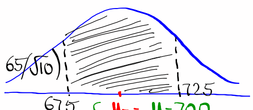
$n=10$

If we randomly select $\boxed{10 \text{ people}}$ find the prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) Cred Score is $\boxed{\text{between } 675 \text{ and } 725.}$

$P(675<\bar{x}<725)$

$= \text{normalcdf}(675, 725, 700, 65/\sqrt{10})$

$= \boxed{.776}$



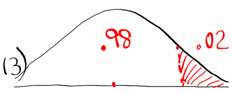
$\text{CLT}\begin{cases}\mu_{\bar{x}} = \mu = 700 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{65}{\sqrt{10}}\end{cases}$

For randomly selected 25 people, find $\bar{x}$, Round to whole #, that separates the top 2% from the rest.

$\bar{x} = P_{98} = \text{invNorm}(.98, 700, 13)$

$= 726.699$

$\approx \boxed{727}$



$\text{CLT}\begin{cases}\mu_{\bar{x}} = \mu = 700 \quad \bar{x} \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{65}{\sqrt{25}} = 13\end{cases}$

Jul 3-10:53 AM

Ages of students are N.D. with $\mu = 32$, and $\sigma = 6$. $N(32, 6)$

$n=2$

If we randomly select $\boxed{2 \text{ students}}$, find the Prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) age is $\boxed{\text{below } 20}$ or $\boxed{\text{above } 35.}$

$P(\bar{x} < 20 \text{ or } \bar{x} > 35)$

$= 1 - \text{normalcdf}(20, 35, 32, 6/\sqrt{2})$

$= \boxed{.242}$

$\text{CLT}\begin{cases}\mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{2}}\end{cases}$

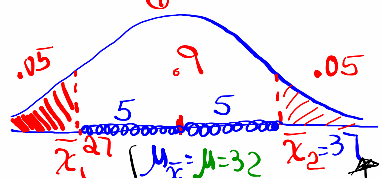
Jul 3-11:08 AM

For randomly selected groups of 4 students, find two $\bar{x}$s, Round to whole #, that separate the $\boxed{\text{middle } 90\%}$ from the rest.

$\bar{x}_1 = P_5 = \text{invNorm}(.05, 32, 3)$

$= 27.065$

$\approx \boxed{27}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = 3 \end{cases}$

$\bar{x}_2 = P_{95} = \text{invNorm}(.95, 32, 3)$

$= 36.935$

$\approx \boxed{37}$

SG 18-21

Jul 3-11:15 AM

Class QZ 16

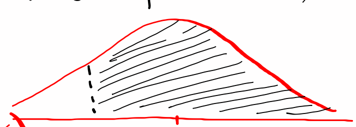
Given $N(150, 18)$

For randomly selected groups of 9,

1) Find $P(\bar{x} > 140)$

$= \text{normalcdf}(140, E99, 150, 6)$

$= \boxed{.952}$



$\mu_{\bar{x}} = \mu = 150$

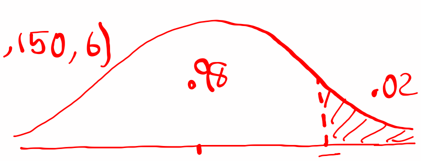
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{9}} = 6$

2) Find $\bar{x} = P_{98}$, Round to 1-decimal.

$\bar{x} = P_{98} = \text{invNorm}(.98, 150, 6)$

$= 162.322$

$\approx \boxed{162.3}$



$\mu_{\bar{x}} = \mu = 150$ $\quad \bar{x}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{9}} = 6$

Jul 3-11:29 AM